

Are seismic waiting time distributions universal?

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We show that seismic waiting time distributions in California and Iceland have many features in common as, for example, a power-law decay with exponent $\alpha \approx 1.1$ for intermediate and with exponent $\gamma \approx 0.6$ for short waiting times. While the transition point between these two regimes scales proportionally with the size of the considered area, the full distribution is not universal and depends in a non-trivial way on the geological area under consideration and its size. This is due to the spatial distribution of epicenters which does *not* form a simple mono-fractal. Yet, the dependence of the waiting time distributions on the threshold magnitude seems to be universal.

1. Introduction: Scaling laws in seismicity

Earthquakes constitute an extremely complex spatio-temporal phenomenon, with the deformation and sudden rupture of some parts of the Earth's crust driven by convective motion in the mantle, and the radiation of energy in the form of seismic waves. Despite this complexity, certain characteristics of seismicity can be captured by simple (empirical) self-similar laws. Thinking of complexity in the sense of the physics of complex systems (e.g. *Gadomski et al.* [2000]), one might argue that the observed scaling laws are in fact the hallmark of the crust being a complex system (see *Mulargia and Geller* [2003] for a recent review). For example, in a specified area covering many geological faults and over a given (large) time window the number of earthquakes $N(m > m_{th})$ with a magnitude m larger than some threshold m_{th} is given by the Gutenberg-Richter law (GR) [*Gutenberg and Richter*, 1949] which states that $\log_{10} N(m > m_{th}) \propto -b m_{th}$ with $b \approx 1$ [*Frohlich and Davis*, 1993]. Taking into account that the strain released during an earthquake is directly related to the moment M of the earthquake (by definition $M = \mu A \delta_e$ where μ is the shear modulus of the rock, A the area of the fault break, and δ_e is the mean displacement across the fault [*Turcotte*, 1997]) which in turn increases exponentially with the magnitude ($\log_{10} M = cm + d$ with $c = 1.5$, $d = 10.73$ [*Stein and Wysession*, 2002]), the probability distribution of the released strain turns out to be a power law, precisely the imprint of scale-free behavior.

This self-similarity exhibited by GR, the independence of stress drop on magnitude [*Ide and Beroza*, 2001; *Kagan*, 2002], and recent laboratory measurements of coseismic slip resistance [*Toro et al.*, 2004] are good evidence that the physics of earthquake rupture is the same for small and large earthquakes. It is still controversial if this is different

for the largest earthquakes (see [*Kagan*, 2002] for a discussion) because of the finite thickness of the brittle layer.

Another example of a self-similar law in seismicity is Omori's law [*Omori*, 1894]. It states that immediately following a main shock there is a sequence of aftershocks whose frequency n decays with time t after the main shock as $n(t) = \frac{C}{(K+t)^p}$ for $t < t_{cutoff}$ with $p \approx 1$ and fault-and-magnitude-dependant constants K , C and t_{cutoff} [*Utsu et al.*, 1995]. An obvious difficulty with Omori's law is the identification of aftershocks especially because aftershocks are not caused by a different relaxation mechanism than main shocks [*Houghs and Jones*, 1997; *Helmstetter and Sornette*, 2003]. Very recently, *Baiesi and Paczuski* [2004] proposed a solution based on a metric measuring the correlation between any two earthquakes.

While the existence and limitations of GR and Omori's law are well tested and accepted, much less is known about other earthquake statistics and their (universal) properties. Here, we will focus on the distribution of time intervals between successive earthquakes above a certain magnitude in a given area which is an important quantity characterizing earthquake occurrence. In the past, many different possibilities have been proposed for these waiting times, from totally random to periodic occurrence of large earthquakes. The most extended view is that of two separated processes, one for main shocks, which ought to follow a Poisson distribution [*Gardner and Knopoff*, 1974] (or not [*Smalley et al.*, 1987; *Sornette and Knopoff*, 1997; *Wang and Kuo*, 1998]), and an independent process to generate aftershocks. Very recently, *Bak et al.* [2002] suggested that waiting time distributions can be described by a power-law with a cutoff for large waiting times which scales with the size of the considered area and magnitude threshold, indicating a generalized scaling behavior. *Corral* [2003] further proposed that the "cutoff" is rather a transition between two different power laws.

Here, we show that yet another power law regime exists for small waiting times. Our results also provide clear evidence that the additional transition point scales differently with area size. We attribute this to the generic structure of the epicenters' spatial distribution which is shown not to form a simple fractal but to have a rather complicated structure, possibly multifractal as proposed in [*Hirabayashi et al.*, 1992; *Goltz*, 1998].

2. Waiting time distributions in California and Iceland

To analyze the distribution of waiting times, we follow [*Bak et al.*, 2002] and take the perspective of statistical physics by neglecting tectonic features and any classification of earthquakes as main shocks or aftershocks. We consider spatial areas and their subdivision into square cells of length L in kilometers. For each of these cells, only events with magnitude above a certain threshold m_{th} are taken into account. In this way, we obtain the waiting times $\tau_i = t_{i+1} - t_i$

in seconds between successive events at time t_i and t_{i+1} in each L^2 -cell. Combining the waiting times from all cells, the probability density function of the waiting times $P_{m_{th},L}(\tau)$ can be estimated.

Analyzing data from southern California, *Bak et al.* [2002] proposed the following scaling law

$$P_{m_{th},L}(\tau) = \tau^{-\alpha} f(\tau L^{d_f}/S^\beta) \quad (1)$$

with $\alpha \simeq 1$, $d_f \simeq 1.2$, $\beta \simeq 1$, $S = 10^{m_{th}}$ and a scaling function f depending only on the combined argument $L^{d_f} S^{-\beta} \tau$. For small arguments, f is approximately constant not affecting the power-law ($1/\tau$) behavior; for large arguments, f decays rapidly and in such a way that it seems to be consistent with a different power-law [Corral, 2003]. While there are good arguments that $\beta \equiv b$ [Corral, 2003], α does not correspond to Omori's p as originally claimed by *Bak et al.* [2002] since the decay of the rate of aftershocks determines the corresponding *all-return time* distribution but not the waiting (or first-return) time distribution. It is an open question if d_f in Eq. 1 corresponds to the fractal dimension D_0 of the epicenter distribution: Corral [2003] has found $D_0 = 1.6$ for southern California. It is also unclear if the analysis by *Bak et al.* [2002] has suffered from the incompleteness of their data for small magnitudes [see Wiemer and Wyss, 2000].

To test the general validity of the proposed scaling law (Eq. 1) and to clarify the scaling with L , we study two earthquake catalogues — one from southern California (\mathcal{C} , 19158 events) and one from southern Iceland (\mathcal{I} , 16286 events). For \mathcal{C} , the coordinates of the polygon are $(120.5^\circ W, 115.0^\circ W) \times (32.5^\circ N, 36.0^\circ N)$. Based on Wiemer and Wyss [2000], the reporting of events is assumed to be homogeneous from January 1984 to December 2000 and complete at the level of $m_c = 2.4$. For \mathcal{I} , it was found that within $(21.43^\circ W, 19.8^\circ W) \times (63.62^\circ N, 64.30^\circ N)$ the reporting of events was homogeneous from July 1991 to December 1995 and complete at the level of $m_c = 0.5$ [Wyss, 2003]. The average waiting time differs by a factor of ≈ 4 from \mathcal{I} to \mathcal{C} .

For \mathcal{C} , we find that indeed $P_{m_{th},L}(\tau)$ decays as a power law with exponent $\alpha \approx 1.05$ over some range of τ . This can be deduced from Fig. 1 and Fig. 2 where $P_{m_{th},L}(\tau)$ is plotted in terms of rescaled coordinates for different values of L . The x -axis is chosen as $x = L^{d_f} S^{-\beta} \tau$ with $S = 10^{m_{th}}$, and the y -axis represents $y = \tau^\alpha P_{m_{th},L}(\tau)$. Thus, the constant regime in Fig. 1 and Fig. 2 corresponds to a power-law decay of $P_{m_{th},L}(\tau)$ with exponent α .

Yet, Fig. 1 and Fig. 2 also show that there is another, previously unnoticed power-law regime for smaller values of x : The power-law increase for the rescaled coordinates implies that the waiting time distributions decay with exponent $\gamma \approx 0.64$. Fig. 1 further shows that there is a rather sharp transition between the two power-law regimes characterized by α and γ , respectively. Moreover, for $d_f = 1.0$, all data collapse nicely onto a single well-defined curve for $x < 10^6$ and over 6 orders of magnitude including the transition points. This data collapse implies in particular that the location of the transition point T_1 between the α - and γ -regime depends on τ , m_{th} and L only through the combined variable x with $d_f = 1.0$. For example, keeping $m_{th} = 2.4$ fixed, we find $T_1 \approx 1.66$ h for $L = 10$ km and $T_1 \approx 6.65$ h for $L = 2.5$ km. Extrapolating to $m_{th} = 5$, for instance, it follows $T_1 \approx 49.0$ h for $L = 100$ km.

However, the data do not collapse onto a single curve for $x > 10^6$ (see Fig. 1) implying that no single scaling function f exists. Thus, the proposed scaling law given in Eq. 1 cannot be strictly valid. This is further confirmed by varying d_f . Already for $d_f = 1.2$ which was used in *Bak et al.* [2002], the data do not collapse onto a single curve for small

values of x . For $d_f \equiv D_0 = 1.6$, the situation is even worse as Fig. 2 shows. Yet, with the exception of the curves for the smallest values of L , $L = 2.5$ km and $L = 5$ km, the data seem to collapse for $x > 4 \times 10^3$ and over 5 orders of magnitude. Since the estimates of $P_{m_{th},L}(\tau)$ for large arguments and small values of L might suffer from insufficient length of the data set leading to an underestimation of the actual y values, a well-defined cutoff or transition point T_2 between the power-law regime characterized by α and the regime for larger arguments showing a rapid decay (which might or might not be a power law) might exist. In contrast to T_1 , T_2 depends on the combined variable x with a different d_f , namely $d_f = 1.6$, and, thus, T_1 and T_2 scale differently with L . Consequently, differences in L are important and cannot be captured by a single exponent d_f .

For \mathcal{I} , we obtain similar results which are shown in Fig. 3 together with data from \mathcal{C} for comparison: For $d_f = 1.0$ and $\beta = 0.95$ and for $x < 10^5$, the different curves collapse nicely over 5 orders of magnitude despite large variations in L and m_{th} and their different geographical origin. In particular, within the statistical errors, there are no differences between \mathcal{I} and \mathcal{C} in terms of α as well as γ . This suggests that $\alpha \approx 1.1$ and $\gamma \approx 0.6$ are universal. The data collapse further confirms that $P_{m_{th},L}(\tau)$ depends on m_{th} only through the combined argument $S^{-\beta} \tau$ implying that a change in m_{th} leads only to a rather simple rescaling of τ . Besides, both for \mathcal{I} and \mathcal{C} we find $\beta \approx b$. Thus, $P_{m_{th},L}(\tau)$ scales with m_{th} and according to GR:

$$P_{m_{th},L}(\tau) = \tilde{P}_L(\tau/10^{b m_{th}}). \quad (2)$$

Here, \tilde{P}_L does not only depend on L but also on the specific geographical area as Fig. 3 shows. Significant differences for large values of x between the curves with the same L but from different geographical regions are present. The waiting time distributions from \mathcal{C} show a very sharp transition at and a much steeper decay for large arguments than the ones from \mathcal{I} . These findings are evidence that even a modified version of Eq. 1 — incorporating the additional exponent γ , for example — with a *universal* function f does not exist. The data from \mathcal{I} , similar to those from \mathcal{C} , also do not collapse for large x despite identical m_{th} . Thus, a simple scaling of $P_{m_{th},L}(\tau)$ with L involving a single dimension d_f does generally not exist. As we show below, this is due to the fact that the distribution of epicenters does not form a simple fractal.

3. Distribution of epicenters in California

Any homogeneous (mono-)fractal is completely described by the capacity dimension D_0 alone. In the case of heterogeneous (multi-)fractals, D_0 describes only one, albeit dominant, feature of the set as can be seen from the generalized definition of fractal dimension (the spectrum of generalized dimensions [Hentschel and Procaccia, 1983]) $D_q = \lim_{L \rightarrow 0} \left(\frac{1}{q-1} \frac{\ln M_q(L)}{\ln L} \right)$. Here, $M_q(L) = \sum_i P_i(L)^q$ are the generalized q th moments of the probabilities $P_i(L) = N_i(L)/N$. $N_i(L)$ is the number of events found in the i th cell of size L , and N is the total number. D_0 is recovered for $q = 0$. Two other prominent fractal dimensions, the information and correlation dimension, result for $q \rightarrow 1$ and $q = 2$, respectively. Higher moments increasingly emphasize densely populated, i.e. seismically more active, areas of the set. D_q may in principle be obtained for any value of q , especially desirable for negative q which would characterize the scaling properties of regions of low seismic activity. Yet, due to the relative sparsity of earthquake data

and its relative inaccuracy, we restrain our analysis to $q \geq 0$ which will nevertheless allow us to judge the non-uniformity of the epicenter distribution. To obtain most reliable results, we base our numerical approach on a generalization of the correlation-integral method as proposed by Pawelzik and Schuster [1987].

Figure 4 shows D_q vs. q for \mathcal{C} and for $0 \leq q \leq 15$. Inset is a double-logarithmic plot of $M_q(L)^{1/(q-1)}$ vs. L for the first three generalized dimensions and for $q = 15$, the highest moment considered. D_q is estimated from the respective slopes by fitting a straight line over the appropriate scaling region which ranges from about 1.5 to 180 km in this case, sufficiently large to believe in the fractal nature of the data. The scaling region holds well even for $q = 15$. We find $D_0 = 1.60 \pm 0.13$, in agreement with Corral [2003]. Furthermore, $D_1 = 1.38 \pm 0.03$, $D_2 = 1.22 \pm 0.05$ and, effectively, $D_\infty \approx 1$. D_∞ is a measure for the fractal dimension of the most densely populated vicinities in the seismicity distribution. Most notably, D_0 and D_∞ are identical within the statistical errors to the values of d_f for T_2 and T_1 , respectively. This suggests that $P_{m_{th},L}(\tau)$ is dominated by the most densely populated vicinities for small waiting times and the dominant feature of the spatial distribution of epicenters captured by the fractal dimension D_0 only prevails for larger waiting times. This is exactly what is expected based on the associated difference in the local rates of seismic activity. Taking $D_\infty - D_0 \approx 0.6$ as a measure of inhomogeneity, we conclude that the seismicity distribution in \mathcal{C} is definitely heterogeneous and, thus, does not form a simple mono-fractal. For \mathcal{I} , we reach the same conclusion.

4. Discussion and conclusions

The quality of our estimate of $P_{m_{th},L}(\tau)$ depends on several aspects: first, on the completeness of the earthquake catalogue. Even if a catalogue is considered to be complete above m_c , certain events are missing: Directly after a large earthquake many small events are lost in the seismic coda of the proceeding event. Hence, short waiting times will be underestimated and the error in τ_i can be as large as the maximum “deadtime” after an earthquake. We, thus, have excluded waiting times less than one minute.

Second, the magnitude of a given earthquake can vary as comparisons of different definitions of magnitude show. Yet, a detailed analysis of the data from \mathcal{I} shows that $P_{m_{th},L}(\tau)$ does not depend significantly on the definition of magnitude chosen.

Third, while errors in the epicenter location can certainly lead to a wrong cell association and, thus, induce errors in the estimate of $P_{m_{th},L}(\tau)$, the (implicit) assumption that earthquakes are a point process is much more severe. As shown by Kanamori and Anderson [1975], it is a good approximation to relate the moment of an earthquake to the area of rupture by $M \propto A^{3/2}$. Thus, A increases with the magnitude of the earthquake; for example, $A^{1/2} \approx 8\text{ km}$ for $m = 6$ and $A^{1/2} \approx 80\text{ km}$ for $m = 8$ [Turcotte, 1997]. Fortunately, for the vast majority of earthquakes $A^{1/2}$ is significantly less than the values of L studied here. This is especially true for \mathcal{I} since the rate of occurrence of the largest earthquakes is rather small, also preventing domination of the statistics by large events.

We are, thus, confident that our estimate of $P_{m_{th},L}(\tau)$ is reliable and the scaling regime with exponent γ for arguments smaller than T_1 is not an artifact of our analysis. This is confirmed by the fact that the scaling of T_1 and T_2 with L can be directly related to the spectrum of generalized dimensions, namely D_∞ and D_0 , respectively. While this directly implies that the proposed scaling law given in Eq. 1 is not valid, the comparison between vastly different

scales and different geological areas suggests that α , γ and T_1 are universal. Our results further suggest that even if a power-law regime exists for arguments larger than T_2 as proposed in [Corral, 2003], it might not be universal. It is up to future studies if the transition from γ to α is associated with aftershock activity.

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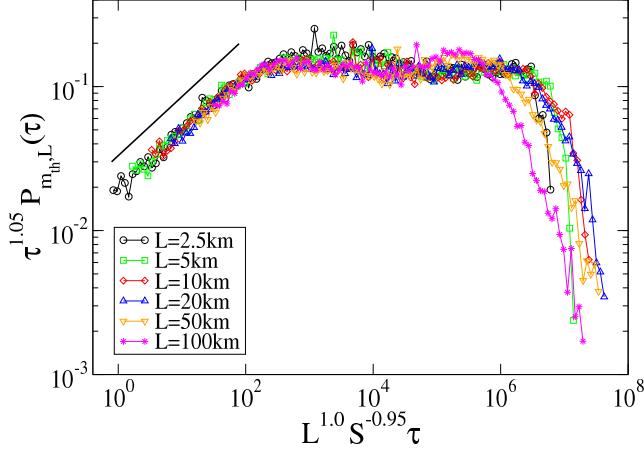


Figure 1. (color online) Rescaled waiting time distributions for California with $\alpha = 1.05$, $d_f = 1.0$, $S = 10^{m_{th}}$, $m_{th} = 2.4$, $\beta = 0.95$. The solid line corresponds to a power-law decay of $P_{m_{th},L}(\tau)$ with exponent $\gamma = 0.64$.

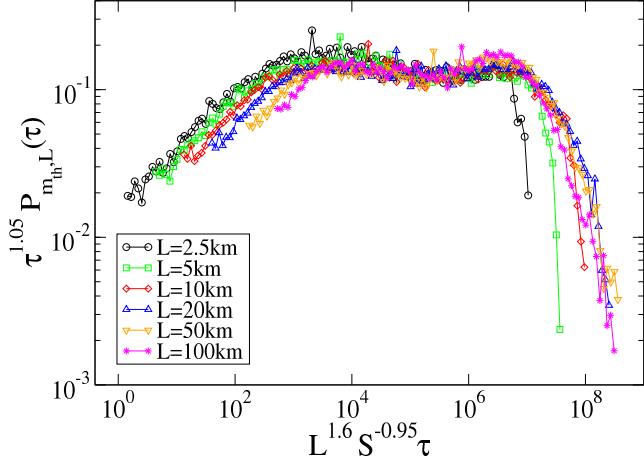


Figure 2. (color online) Rescaled waiting time distribution for California as in Fig. 1 but with $d_f = 1.6$.

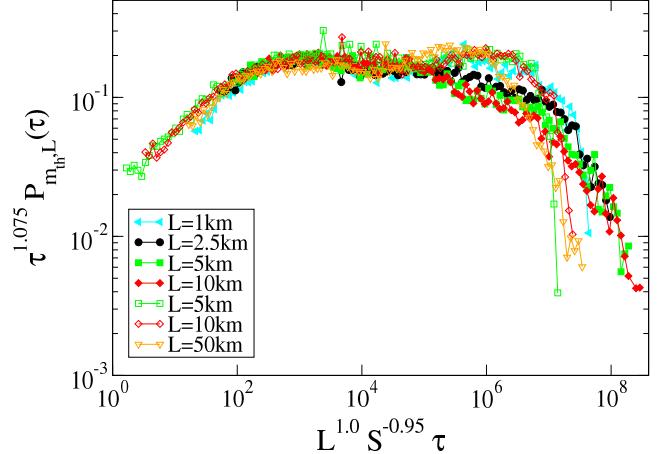


Figure 3. (color online) Combined rescaled waiting time distributions for Iceland (solid symbols, $m_{th} = 0.5$) and California (open symbols, $m_{th} = 2.4$) with $\alpha = 1.075$, $\beta = 0.95$, $d_f = 1.0$, $S = 10^{m_{th}}$. Since the average error in epicenter locations is smaller for Iceland ($< 1\text{km}$) than for California ($\approx 1.7\text{km}$), we have used different values for L .

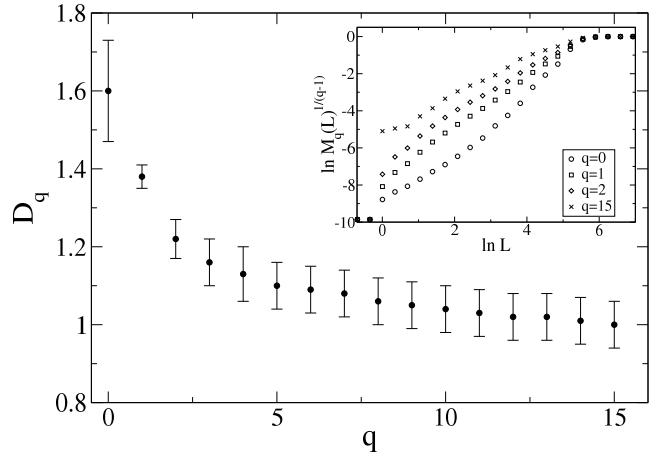


Figure 4. Spectrum of generalized dimensions D_q for the epicenter distribution of California. Inset: generalized moments, from which D_q is obtained.